Enrollment No: ____________________ **Exam Seat No:**____________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Differential Equations

Subject Code : 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester : 1 **Date : 26/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **(07)**

- **a.** Find $\Gamma(\frac{1}{2})\,\Gamma(\frac{5}{2})$. **(01)**
- **b.** State Rodrigue's formula. **(01)**
- **c.** State Picard's theorem. **(01)**
- **d.** Solve: $(6x + yz)dx + (xz - 2y)dy + (xz + 2z)dz = 0$ **(01)**
- **e.** The degree of differential equation $y'' - \sqrt{y'} = 0$ is _______ **(01)**
- **f.** Find $P_0(x)$ and $P_n(-1)$. **(01)**
- **g.** Evaluate: $\int_{-1}^{1} x^6\, P_8(x)dx$ **(01)**

Q-2 **Attempt all questions** **(14)**

- **a.** Find the general solution of the equation $(D^2 + 2D + 1)y = 3x^{\frac{3}{2}}\, e^{-x}$ using the method of variation of parameters. **(06)**
- **b.** Show that $x = 0$ is a regular singular point and $x = 1$ is an irregualar singular point of the equation : $x(x-1)^3y'' + 2(x-1)^3y' + 3y = 0$. **(05)**
- **c.** Find the radius of convergence of the following series: **(03)**
  - i) $\sum \frac{n}{(n+1)^2} x^2$
  - ii) $\sum \frac{x^n}{(n)^n}$

OR

Q-2 **Attempt all questions** **(14)**

- **a.** Obtain power series solution of the form $\sum a_n x^n$ for the differential equation $(x^2 + 1)y'' + 4xy' + 6y = 0$ at $x_0 = 0$. **(06)**
- **b.** Prove : i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ and ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ **(06)**
- **c.** Determine the nature of the point $x = 0$ for the equation, **(02)**

$xy'' + y\sin x = 0.$

Q-3 **Attempt all questions** **(14)**

a. If $f(x) = \begin{cases} 0 & -1 \le x < 0 \\ x & 0 \le x < 1 \end{cases}$ then find the fourier legendre expansion of f. **(06)**

b. Show that (i) $J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$ **(05)**

(ii) $J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$

c. Show that $J_p(ax)$ is solution of $y'' + \frac{1}{x}y' + \left(a^2 - \frac{p^2}{x^2}\right)y = 0$ **(03)**

OR

Q-3 **Attempt all questions** **(14)**

a. Show that $(1 - 2xt + t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$ $where$ $|x| \le 1, |t| < 1.$ **(07)**

b. Prove the following Recurrance relation : **(07)**

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x) \quad \forall\, n \ge 2.$$

SECTION – II

Q-4 **Attempt the Following questions.** **(07)**

a. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ **(01)**

b. Solve: $p = e^q$ **(01)**

c. Find $F(1; 1; 2; x)$. **(01)**

d. $J_n(x)$ and $J_{(-n)}(x)$ are linearly dependent. Determine whether the statement is true or false. **(01)**

e. State the necessary and sufficient condition that Pfaffian differential equation in three variables is integral. **(01)**

f. Solve: $p^2 - q^2 - x + y = 0.$ **(01)**

g. Hypergeometric function is symmetric. Determine whether the statement is true or false. **(01)**

Q-5 **Attempt all questions** **(14)**

a. Using Picard's method of successive approximation ,find three approximation of the solution of the equation $\frac{dy}{dx} = 2y - 2x^2 - 3$,where $y = 2$ when $x = 0$. **(05)**

b. Prove: (i) $\log(1+x) = x\,F(1; 1; 2; -x)$ **(05)**

(ii) $(1+x)^n = F(-n; 1; 1; -x)$

c. Show that the equations $xp - yq = 0$ and $xzp + yzq - 2xy = 0$ are compatible. **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a. Solve : $y^2p - xyq = x(z - 2y)$ **(05)**

b. Find the general integral of $yzp + xzq = xy$. **(05)**

c. Eliminate the arbitrary functions and hence obtain partial differential equation: (i) $z = F(x^2 - y^2)$ and (ii) $z = e^y F(x + y)$ **(04)**

Q-6 **Attempt all questions** **(14)**

a. Find the complete integral of $(z + px)^2 - q = 0$ using Charpit's method. **(07)**

b. Prove that if X is a vector such that $X \cdot curl\, X = 0$ then $\mu X \cdot curl\, \mu X = 0$ where μ is an arbitrary function of x, y, z. **(04)**

c. Show that the following Pfaffian differential equation is integrable: **(03)**

$$y\, dx + x\, dy + 2z\, dz = 0$$

OR

Q-6 **Attempt all Questions** **(14)**

a. Verify that the Pfaffian differential equation is integrable and find corresponding solution: **(07)**

$$x^2y^2z^2\, dx + b^2x^2z^2\, dy + c^2x^2y^2\, dz = 0$$

b. Using Jacobi's method ,solve the partial differential equation: **(07)**

$${u_x}^2 + {u_y}^2 + u_z - 1 = 0$$